Reflector design

This problem concerns the existence and regularity of the surface $M$ (see picture below) such that the light from $o$ is reflected to the surface $\Sigma$. By computing the Jacobian of the mapping $T : X \in U \rightarrow T(X) \in \Sigma$, we get a Monge-Ampere type equation for $\rho$ on the sphere, subject to the boundary condition $T(\partial U) = \partial \Sigma$.

The far field case. That is when $\Sigma$ is at infinity. We consider instead the mapping $T : X \in U \rightarrow Y \in V$. In [rd1] we proved the existence, uniqueness, and (interior) regularity of solutions to the problem [rd1]. In [rd2] we proved that the reflector design is an optimal transportation. Therefore the linear programming can be used to obtain a numerical solution to the problem (except when $M$ is rotationally symmetric, it was not at all clear how to get a satisfactory algorithm for a Monge-Ampere type equation with the special boundary condition $T(\partial U) = \partial V$).

Some ideas in [rd1] have been used in our work on the regularity of optimal transportation.

The near field case (the general case). In [rd3] we proved the existence and (interior) regularity of solutions to the problem. The near field case is much more complicated.

We would like to point out that when $\Sigma$ lies in a plane passing through the origin, $M$ satisfies the standard Monge-Ampere equation [rd3].

References